Table 2.1 Fourier-transform theorems

Property	Mathematical description
1. Linearity	$ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where a and b are constants
2. Dilation	$g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$
4. Time shifting	$g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
5. Frequency shifting	$g(t)\exp(-j2\pi f_0 t) \rightleftharpoons G(f-f_0)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t)\mathrm{d}t = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f)\mathrm{d}f$
8. Differentiation in the time domain	$\frac{\mathrm{d}}{\mathrm{d}t}g(t) \rightleftharpoons j2\pi f G(f)$
9. Integration in the time domain	$\int_{-\infty}^{t} g(\tau)\mathrm{d}\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$
11. Multiplication in the time domain	$g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\mathrm{d}\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{t} g_1(\tau)g_2(t-\tau)\mathrm{d}\tau \rightleftharpoons G_1(f)G_2(f)$
13. Correlation theorem	$\int_{-\infty}^{\infty} g_1(t)g_2^*(t-\tau)\mathrm{d}\tau \rightleftharpoons G_1(f)G_2^*(f)$
14. Rayleigh's energy theorem	$\int_{-\infty}^{\infty} \lvert g(t)\rvert^2\mathrm{d}t = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2\mathrm{d}f$
15. Parseval's power theorem for periodic signal of period T_0	$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \lvert g(t)\rvert^2\mathrm{d}t = \sum_{n=-\infty}^{\infty} \lvert G(f_n)\rvert^2, \quad f_n = n/T_0$

Table 2.2 Fourier-transform pairs and commonly used time functions

Time function	Fourier transform
1. $\mathrm{rect}\left(\frac{t}{T}\right)$	$T\,\mathrm{sinc}(fT)$
2. $\mathrm{sinc}(2Wt)$	$\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$
3. $\exp(-at)u(t), \quad a>0$	$\frac{1}{a+\mathrm{j}2\pi f}$
4. $\exp(-a\lvert t\rvert), \quad a>0$	$\frac{2a}{a^2+(2\pi f)^2}$
5. $\exp(-\pi t^2)$	$\exp(-\pi f^2)$
6. $\begin{cases} 1-\frac{\lvert t\rvert}{T}, & \lvert t\rvert<T \\ 0, & \lvert t\rvert \ge T \end{cases}$	$T\,\mathrm{sinc}^2(fT)$
7. $\delta(t)$	1
8. 1	$\delta(f)$
9. $\delta(t-t_0)$	$\exp(-\mathrm{j}2\pi f t_0)$
10. $\exp(\mathrm{j}2\pi f_c t)$	$\delta(f-f_c)$
11. $\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)]$
12. $\sin(2\pi f_c t)$	$\frac{1}{2\mathrm{j}}[\delta(f-f_c)-\delta(f+f_c)]$
13. $\mathrm{sgn}(t)$	$\frac{1}{\mathrm{j}\pi f}$
14. $\frac{1}{\pi t}$	$-\mathrm{j}\,\mathrm{sgn}(f)$
15. $u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{\mathrm{j}2\pi f}$
16. $\sum_{i=-\infty}^{\infty}\delta(t-iT_0)$	$f_0\sum_{n=-\infty}^{\infty}\delta(f-nf_0), \quad f_0=\frac{1}{T_0}$

Definitions

Unit step function:

$$u(t) = \begin{cases} 1, & t>0 \\ \frac{1}{2}, & t=0 \\ 0, & t<0 \end{cases}$$

Dirac delta function:

$\delta(t) = 0$ for $t \neq 0$ and

$$\int_{-\infty}^{\infty}\delta(t)\,\mathrm{d}t = 1$$

Rectangular function:

$$\mathrm{rect}(t) = \begin{cases} 1, & -\frac{1}{2} < t \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Signum function:

$$\mathrm{sgn}(t) = \begin{cases} +1, & t>0 \\ 0, & t=0 \\ -1, & t<0 \end{cases}$$

Sinc function:

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

Gaussian function:

$$g(t) = \exp(-\pi t^2)$$